Interaction of Non-Abelian Tensor Gauge Fields

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Abstract

Recently we introduced an extended vector bundle X on which non-Abelian tensor gauge fields realize a connection. Our aim here is to introduce interaction of non-Abelian tensor gauge fields with fermions and bosons. We have found that there exist two series of gauge invariant forms describing this interaction. The linear sum of these forms comprises the general gauge invariant Lagrangian. Studying the corresponding Euler-Lagrange equations we found that a particular linear combination of these forms exhibits enhanced symmetry which guarantees the conservation of the corresponding high-rank currents. A possible mechanism of symmetry breaking and mass generation of tensor gauge bosons is suggested.

1 Introduction

It is appealing to extend Yang-Mills theory [1, 2] so that it will define the interaction of fields which carry not only non-commutative internal charges, but also arbitrary large spins. This extension will induce the interaction of matter fields mediated by charged gauge quanta carrying spin larger than one [3]. In our recent approach these gauge fields are defined as rank-$(s+1)$ tensors [3, 4, 5, 6]

$$A^a_{\mu\lambda_1...\lambda_s}(x)$$

and are totally symmetric with respect to the indices $\lambda_1...\lambda_s$. A priory the tensor fields have no symmetries with respect to the first index μ. The index s runs from zero to infinity. The first member of this family of the tensor gauge bosons is the Yang-Mills vector boson A^a_μ. This is an essential departure from the previous considerations, in which the higher-rank tensors were totally symmetric [7, 8, 9, 10, 11, 14, 15, 16, 17, 18].

The extended non-Abelian gauge transformation of the tensor gauge fields [3, 4, 5] is defined by the equation (7) and comprises a closed algebraic structure, because the commutator of two transformations can be expressed in the form

$$[\,\delta_\eta, \delta_\xi]\, A_{\mu\lambda_1\lambda_2...\lambda_s} \;=\; -\,ig\,\, \delta_\zeta A_{\mu\lambda_1\lambda_2...\lambda_s},$$

where the gauge parameters $\{\zeta\}$ are given by the matrix commutators (9). This allows to define generalized field strength tensors (13) $G^a_{\mu\nu,\lambda_1...\lambda_s}$ which are *transforming homogeneously* (14) with respect to the extended gauge transformations (7). The field strength tensors $G^a_{\mu\nu,\lambda_1....\lambda_s}$ are used to construct two infinite series of gauge invariant quadratic forms

$$\mathcal{L}_s \;\;,\;\; \mathcal{L}'_s \qquad s = 2, 3, ...$$

Each term of these infinite series is separately gauge invariant with respect to the extended gauge transformations (7). These forms contain quadratic kinetic terms and terms describing nonlinear interaction of Yang-Mills type. In order to make all tensor gauge fields dynamical one should add all these forms together. Thus the gauge invariant Lagrangian describing dynamical tensor gauge bosons of all ranks has the form [3, 4, 5]

$$\mathcal{L} = \sum_{s=1}^{\infty} g_s \mathcal{L}_s \;+\; \sum_{s=2}^{\infty} g'_s \mathcal{L}'_s \;, \tag{1}$$

where $\mathcal{L}_1 \equiv \mathcal{L}_{YM}$ is the Yang-Mills Lagrangian.

It is important that: i) *the Lagrangian does not contain higher derivatives of tensor gauge fields ii) all interactions take place through the three- and four-particle exchanges with dimensionless coupling constant g iii) the complete Lagrangian contains all higher-rank tensor gauge fields and should not be truncated iv) the invariance with respect to the extended gauge transformations does not fix the coupling constants g_s and g'_s.*

The coupling constants g_s and g'_s remain arbitrary because every term of the sum is separately gauge invariant and the extended gauge symmetry alone does not fix them. There is a freedom to vary these constants without breaking the extended gauge symmetry (7). The main point here is that one can achieve the enhancement of the extended gauge symmetry properly tuning the coupling constants g_s and g'_s. Indeed, considering a linear sum of two gauge invariant forms in (1)

$$g_2\mathcal{L}_2 + g'_2\mathcal{L}'_2,$$

which describe the rank-2 tensor gauge field $A^a_{\mu\lambda}$, we found [3, 5] that for

$$g'_2 = g_2$$

the sum $\mathcal{L}_2 + \mathcal{L}'_2$ exhibits invariance with respect to a bigger gauge group (18). In addition to the extended gauge group (7), which we had initially, we get a bigger gauge group with double number of gauge parameters [3, 5, 6]. Considering the second pair of quadratic forms in (1)

$$g_3\mathcal{L}_3 + g'_3\mathcal{L}'_3$$

which describe the rank-3 tensor gauge field $A^a_{\mu\lambda\rho}$, we found in [19] that for

$$g'_3 = \frac{4}{3}g_3$$

the system also has an enhanced gauge symmetry (18). The explicit description of these symmetries together with the corresponding field equations is given in [19].

Our aim now is to extend this construction to a system of interacting tensor gauge fields with higher-spin fermion and boson fields. The fermions are defined as Rarita-Schwinger spinor-tensors [20, 21, 22]

$$\psi^\alpha_{\lambda_1...\lambda_s}(x)$$

with mixed transformation properties of Dirac four-component wave function (the index α denotes the Dirac index) and are totally symmetric tensors of the rank s over the indices $\lambda_1...\lambda_s$. All fields of the $\{\psi\}$ family are isotopic multiplets belonging to the same representation σ of the compact Lie group G (the corresponding indices are suppressed). The bosons are defined as totally symmetric Fierz-Pauli rank-s tensors [8]

$$\phi_{\lambda_1...\lambda_s}(x)$$

all belonging to the same representation τ of the compact Lie group G.

We shall demonstrate that the gauge invariant Lagrangian for fermions and bosons also contains two infinite series of quadratic forms and the general Lagrangian is a linear sum of these forms. For fermions it takes the form

$$\mathcal{L}^F = \sum_{s=0}^{\infty} f_s \, \mathcal{L}_{s+1/2} + \sum_{s=1}^{\infty} f'_s \, \mathcal{L}'_{s+1/2} \tag{2}$$

and for bosons it is

$$\mathcal{L}^B = \sum_{s=0}^{\infty} b_s \, \mathcal{L}^B_s + \sum_{s=1}^{\infty} b'_s \, \mathcal{L}'^B_s. \tag{3}$$

Again it is important to notice that the invariance with respect to the extended gauge transformations does not fix the coupling constants f_s, f'_s and b_s, b'_s. The coupling constants f_s, f'_s and b_s, b'_s remain arbitrary. Every term of the sum is separately gauge invariant and the extended gauge symmetry alone does not define them. *The basic principle which we shall pursue in our construction will be to fix these coupling constants demanding realization of enhanced symmetries and unitarity of the theory*[1].

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[1]For that one should study the spectrum of the theory and its dependence on these coupling constants. For some particular values of coupling constants the linear sum of these forms may exhibit symmetries with respect to a bigger gauge group $\mathcal{G} \supset G$.

In the second section we shall outline the transformation properties of non-Abelian tensor gauge fields, the definition of the corresponding field stress tensors, the general expression for the invariant Lagrangian and its enhanced symmetries [3, 4, 5]. In the third, forth and fifths sections we shall incorporate into the theory fermions of half-integer spins. We shall construct two infinite series of gauge invariant forms (2). The invariant Lagrangian is a linear sum of all these forms and describes interaction of non-Abelian tensor gauge fields with half-integer spin fermions. At special values of the coupling constants it shows up enhanced symmetries and therefore defines conserved tensor currents. In the sixth, seventh and eighth sections the above construction will be extended to include integer-spin boson fields and a possible symmetry breaking mechanism to generate masses of tensor gauge bosons is suggested.

2   Non-Abelian Tensor Gauge Fields

The gauge fields are defined as rank-$(s + 1)$ tensors [3]

$$A^a_{\mu\lambda_1...\lambda_s}(x), \qquad s = 0, 1, 2, ...$$

and are totally symmetric with respect to the indices $\lambda_1...\lambda_s$. A priory the tensor fields have no symmetries with respect to the first index μ. The index a numerates the generators L^a of the Lie algebra $\breve{g}$ of a *compact*[2] Lie group G.

One can think of these tensor fields as appearing in the expansion of the extended gauge field $\mathcal{A}_\mu(x, e)$ over the unite vector e_λ [5]:

$$\mathcal{A}_\mu(x, e) = \sum_{s=0}^{\infty} \frac{1}{s!} \, A^a_{\mu\lambda_1...\lambda_s}(x) \, L^a e_{\lambda_1}...e_{\lambda_s}. \tag{4}$$

The gauge field $A^a_{\mu\lambda_1...\lambda_s}$ carries indices $a, \lambda_1, ..., \lambda_s$ labeling the generators of *extended current algebra* $\mathcal{G}$ *associated with compact Lie group* G. It has infinite many generators $L^a_{\lambda_1...\lambda_s} = L^a e_{\lambda_1}...e_{\lambda_s}$ and the corresponding algebra is given by the commutator [5] [3]

$$[L^a_{\lambda_1...\lambda_s}, L^b_{\rho_1...\rho_k}] = i f^{abc} L^c_{\lambda_1...\lambda_s\rho_1...\rho_k}. \tag{5}$$

Because $L^a_{\lambda_1...\lambda_s}$ are space-time tensors, the full algebra includes the Poincaré generators P_μ, $M_{\mu\nu}$. They act on the space-time components of the above generators as follows:

$$[P_\mu, \, P_\nu] = 0,$$
$$[M_{\mu\nu}, \, P_\lambda] = \eta_{\nu\lambda} \, P_\mu - \eta_{\mu\lambda} \, P_\nu,$$
$$[M_{\mu\nu}, \, M_{\lambda\rho}] = \eta_{\mu\rho} \, M_{\nu\lambda} - \eta_{\mu\lambda} \, M_{\nu\rho} + \eta_{\nu\lambda} \, M_{\mu\rho} - \eta_{\nu\rho} \, M_{\mu\lambda},$$
$$[P_\mu, \, L^a_{\lambda_1...\lambda_s}] = 0,$$
$$[M_{\mu\nu}, \, L^a_{\lambda_1...\lambda_s}] = \eta_{\nu\lambda_1} \, L^a_{\mu\lambda_2...\lambda_s} - \eta_{\mu\lambda_1} \, L^a_{\nu\lambda_2...\lambda_s} + ... + \eta_{\nu\lambda_s} \, L^a_{\mu\lambda_1...\lambda_{s-1}} - \eta_{\mu\lambda_s} \, L^a_{\nu\lambda_1...\lambda_{s-1}},$$
$$[L^a_{\lambda_1...\lambda_s}, L^b_{\rho_1...\rho_k}] = i f^{abc} L^c_{\lambda_1...\lambda_s\rho_1...\rho_k}. \tag{6}$$

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[2]The algebra $\breve{g}$ possesses an orthogonal basis in which the structure constants f^{abc} are totally antisymmetric.

[3]See also the alternative Abelian expansions in [10, 11, 23, 24] and the algebras based on diffeomorphisms group in [25, 26].

It is an extension of the Poincaré algebra by generators which contains isospin algebra G. In some sense the new vector variable e_λ plays a role similar to the grassmann variable θ in supersymmetry algebras [12, 13].

The extended non-Abelian gauge transformations of the tensor gauge fields are defined by the following equations [4]:

$$
\begin{aligned}
\delta A_\mu^a &= (\delta^{ab}\partial_\mu + g f^{acb} A_\mu^c)\xi^b, \\
\delta A_{\mu\nu}^a &= (\delta^{ab}\partial_\mu + g f^{acb} A_\mu^c)\xi_\nu^b + g f^{acb} A_{\mu\nu}^c \xi^b, \\
\delta A_{\mu\nu\lambda}^a &= (\delta^{ab}\partial_\mu + g f^{acb} A_\mu^c)\xi_{\nu\lambda}^b + g f^{acb}(A_{\mu\nu}^c \xi_\lambda^b + A_{\mu\lambda}^c \xi_\nu^b + A_{\mu\nu\lambda}^c \xi^b),
\end{aligned}
\tag{7}
$$

$$
.... \quad . \quad ....,
$$

where $\xi_{\lambda_1...\lambda_s}^a(x)$ are totally symmetric gauge parameters. These extended gauge transformations generate a closed algebraic structure. To see that, one should compute the commutator of two extended gauge transformations δ_η and δ_ξ of parameters η and ξ. The commutator of two transformations can be expressed in the form [4]

$$
[\,\delta_\eta, \delta_\xi]\, A_{\mu\lambda_1\lambda_2...\lambda_s} = -ig\, \delta_\zeta A_{\mu\lambda_1\lambda_2...\lambda_s}
\tag{8}
$$

and is again an extended gauge transformation with the gauge parameters $\{\zeta\}$ which are given by the matrix commutators

$$
\begin{aligned}
\zeta &= [\eta, \xi] \\
\zeta_{\lambda_1} &= [\eta, \xi_{\lambda_1}] + [\eta_{\lambda_1}, \xi] \\
\zeta_{\nu\lambda} &= [\eta, \xi_{\nu\lambda}] + [\eta_\nu, \xi_\lambda] + [\eta_\lambda, \xi_\nu] + [\eta_{\nu\lambda}, \xi],
\end{aligned}
\tag{9}
$$

$$
...... \quad . \quad ....
$$

Each single field $A_{\mu\lambda_1...\lambda_s}^a(x),\ s = 2, 3, ...$ has no geometrical interpretation, but all these fields together with $A_\mu^a(x)$ have geometrical interpretation in terms of *connection on the extended vector bundle X* [5]. Indeed, one can define the extended vector bundle X whose structure group is $\mathcal{G}$ with group elements

$$
U(\xi) = exp[\,i\xi(x, e)\,],
$$

where

$$
\xi(x, e) = \sum_s \frac{1}{s!}\, \xi_{\lambda_1...\lambda_s}^a(x)\ L^a e_{\lambda_1}...e_{\lambda_s}.
$$

Defining the extended gauge transformation of $\mathcal{A}_\mu(x, e)$ in a standard way

$$
\mathcal{A}_\mu'(x, e) = U(\xi)\mathcal{A}_\mu(x, e)U^{-1}(\xi) - \frac{i}{g}\partial_\mu U(\xi)\, U^{-1}(\xi),
\tag{10}
$$

we get the extended vector bundle X on which the gauge field $\mathcal{A}_\mu^a(x, e)$ is a connection [2]. The expansion of (10) over the vector e_λ reproduces gauge transformation law of the tensor gauge fields (7). Using the commutator of the covariant derivatives $\nabla_\mu^{ab} = (\partial_\mu - ig\mathcal{A}_\mu(x, e))^{ab}$

$$
[\nabla_\mu, \nabla_\nu]^{ab} = g f^{acb}\mathcal{G}_{\mu\nu}^c,
\tag{11}
$$

we can define the extended field strength tensor

$$
\mathcal{G}_{\mu\nu}(x, e) = \partial_\mu\mathcal{A}_\nu(x, e) - \partial_\nu\mathcal{A}_\mu(x, e) - ig[\mathcal{A}_\mu(x, e)\ \mathcal{A}_\nu(x, e)]
\tag{12}
$$

5

which transforms homogeneously: $\mathcal{G}'_{\mu\nu}(x,e)) = U(\xi)\mathcal{G}_{\mu\nu}(x,e)U^{-1}(\xi)$. Thus the *generalized field strengths are defined as* [4]

$$
\begin{aligned}
G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu, \\
G^a_{\mu\nu,\lambda} &= \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda} + gf^{abc}(A^b_\mu A^c_{\nu\lambda} + A^b_{\mu\lambda} A^c_\nu), \\
G^a_{\mu\nu,\lambda\rho} &= \partial_\mu A^a_{\nu\lambda\rho} - \partial_\nu A^a_{\mu\lambda\rho} + gf^{abc}(A^b_\mu A^c_{\nu\lambda\rho} + A^b_{\mu\lambda} A^c_{\nu\rho} + A^b_{\mu\rho} A^c_{\nu\lambda} + A^b_{\mu\lambda\rho} A^c_\nu),
\end{aligned}
\tag{13}
$$

$$\dots\dots \quad . \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

and transform homogeneously with respect to the extended gauge transformations (7). The field strength tensors are antisymmetric in their first two indices and are totally symmetric with respect to the rest of the indices. The inhomogeneous extended gauge transformation (7) induces the homogeneous gauge transformation of the corresponding field strength (13) of the form [4]

$$
\begin{aligned}
\delta G^a_{\mu\nu} &= gf^{abc}G^b_{\mu\nu}\xi^c, \\
\delta G^a_{\mu\nu,\lambda} &= gf^{abc}(G^b_{\mu\nu,\lambda}\xi^c + G^b_{\mu\nu}\xi^c_\lambda), \\
\delta G^a_{\mu\nu,\lambda\rho} &= gf^{abc}(G^b_{\mu\nu,\lambda\rho}\xi^c + G^b_{\mu\nu,\lambda}\xi^c_\rho + G^b_{\mu\nu,\rho}\xi^c_\lambda + G^b_{\mu\nu}\xi^c_{\lambda\rho}),
\end{aligned}
\tag{14}
$$

$$\dots\dots \quad . \quad \dots\dots\dots\dots\dots\dots\dots\dots$$

The symmetry properties of the field strength $G^a_{\mu\nu,\lambda_1\dots\lambda_s}$ remain invariant in the course of this transformation.

These tensor gauge fields and the corresponding field strength tensors allow to construct two series of gauge invariant quadratic forms. The first series is given by the formula [4]:

$$
\begin{aligned}
\mathcal{L}_{s+1} &= -\frac{1}{4} G^a_{\mu\nu,\lambda_1\dots\lambda_s} G^a_{\mu\nu,\lambda_1\dots\lambda_s} + \dots\dots \\
&= -\frac{1}{4}\sum_{i=0}^{2s} a^s_i G^a_{\mu\nu,\lambda_1\dots\lambda_i} G^a_{\mu\nu,\lambda_{i+1}\dots\lambda_{2s}}(\sum_{p's}\eta^{\lambda_{i_1}\lambda_{i_2}}\dots\dots\eta^{\lambda_{i_{2s-1}}\lambda_{i_{2s}}}) ,
\end{aligned}
\tag{15}
$$

where the sum $\sum_p$ runs over all nonequal permutations of $i's$, in total $(2s-1)!!$ terms and the numerical coefficient is

$$
a^s_i = \frac{s!}{i!(2s-i)!} \ .
$$

The second series of gauge invariant quadratic forms is given by the formula [3, 5]:

$$
\begin{aligned}
\mathcal{L}'_{s+1} &= \frac{1}{4} G^a_{\mu\lambda_1,\lambda_2\dots\lambda_{s+1}} G^a_{\mu\lambda_2,\lambda_1\dots\lambda_{s+1}} + \dots\dots \\
&= \frac{1}{8}\sum_{i=1}^{2s+1} a^s_{i-1} G^a_{\mu\lambda_1,\lambda_2\dots\lambda_i} G^a_{\mu\lambda_{i+1},\lambda_{i+2}\dots\lambda_{2s+2}}(\sum_{p's}'\eta^{\lambda_{i_1}\lambda_{i_2}}\dots\dots\eta^{\lambda_{i_{2s+1}}\lambda_{i_{2s+2}}}) ,
\end{aligned}
\tag{16}
$$

where the sum $\sum'_p$ runs over all nonequal permutations of $i's$, with exclusion of the terms which contain $\eta^{\lambda_1,\lambda_{i+1}}$.

In order to make all tensor gauge fields dynamical one should add the corresponding kinetic terms. Thus the invariant Lagrangian describing dynamical tensor gauge bosons of all ranks has the form

$$
\mathcal{L} = \sum_{s=1}^{\infty} g_s\mathcal{L}_s + \sum_{s=2}^{\infty} g'_s\mathcal{L}'_s ,
\tag{17}
$$

where $\mathcal{L}_1 \equiv \mathcal{L}_{YM}$.

As we already noticed in the Introduction the invariance with respect to the extended gauge transformations does not fix the coupling constants g_s and g_s'. Therefore we can tune these coupling constants demanding maximal possible symmetry of the sum. We found in [5, 19] that the coupling constants should be chosen as $g_2' = g_2$, $g_3' = \frac{4}{3}g_3$. The free part of the Lagrangian is invariant with respect to the large gauge group of transformations with additional gauge parameters $\zeta_\mu^a, \zeta_{\mu\nu}^a$:

$$
\begin{aligned}
\delta A_\mu^a &= \partial_\mu \xi^a, \\
\delta A_{\mu\lambda}^a &= \partial_\mu \xi_\lambda^a + \partial_\lambda \zeta_\mu^a, \\
\delta A_{\mu\nu\lambda}^a &= \partial_\mu \xi_{\nu\lambda}^a + \partial_\nu \zeta_{\mu\lambda}^a + \partial_\lambda \zeta_{\mu\nu}^a,
\end{aligned}
\tag{18}
$$

where the gauge parameters $\zeta_{\mu\lambda}^a$ should fulfil the constraint $\partial_\rho \zeta_{\rho\lambda}^a - \partial_\lambda \zeta_{\rho\rho}^a = 0$. The coupling constants g_2 and g_3 remain arbitrary and define mixing amplitudes between lower- and higher-rank tensor gauge bosons. They have to be fixed by additional physical requirements imposed on these amplitudes. We shall return to this problem later.

3   First Series of Gauge Invariant Forms for Fermions

The fermions are defined as Rarita-Schwinger spinor-tensor fields [20, 21, 22]

$$
\psi_{\lambda_1...\lambda_s}^\alpha(x)
\tag{19}
$$

with mixed transformation properties of Dirac four-component wave function and are totally symmetric tensors of the rank s over the indices $\lambda_1...\lambda_s$ (the index α denotes the Dirac index and will be suppressed in the rest part of the article). All fields of the $\{\psi\}$ family are isotopic multiplets $\psi_{\lambda_1...\lambda_s}^i(x)$ belonging to the same representation σ_{ij}^a of the compact Lie group G (the index i denotes the isotopic index). One can think of these spinor-tensor fields as appearing in the expansion of the extended fermion field $\Psi^i(x, e)$ over the unit tangent vector e_λ [3, 5]

$$
\Psi^i(x, e) = \sum_{s=0}^\infty \psi_{\lambda_1...\lambda_s}^i(x)\ e_{\lambda_1}...e_{\lambda_s}.
\tag{20}
$$

Our intention is to introduce gauge invariant interaction of fermion fields with non-Abelian tensor gauge fields. The transformation of the fermions under the extended isotopic group we shall define by the formula [4]

$$
\Psi'(x, e) = \mathcal{U}(\xi)\Psi(x, e),
\tag{21}
$$

where

$$
\mathcal{U}(\xi) = \exp(ig\xi(x, e)), \quad \xi(x, e) = \sum_{s=0}^\infty \xi_{\lambda_1...\lambda_s}^a(x)\ \sigma^a e_{\lambda_1}...e_{\lambda_s}
$$

and σ^a are the matrices of the representation σ of the compact Lie group G, according to which all $\psi's$ are transforming. In components the transformation of fermion fields under the extended isotopic group therefore will be [4]

$$
\begin{aligned}
\delta_\xi \psi &= ig\sigma^a \xi^a \psi, \\
\delta_\xi \psi_\lambda &= ig\sigma^a(\xi^a\ \psi_\lambda + \xi_\lambda^a\ \psi), \\
\delta_\xi \psi_{\lambda\rho} &= ig\sigma^a(\xi^a\ \psi_{\lambda\rho} + \xi_\lambda^a\ \psi_\rho + \xi_\rho^a\ \psi_\lambda + \xi_{\lambda\rho}\ \psi), \\
&\quad .... \quad . \quad ....,
\end{aligned}
\tag{22}
$$

The covariant derivative of the fermion field is defined as usually:

$$\nabla_\mu \Psi = i\partial_\mu \Psi + g\mathcal{A}_\mu(x,e)\Psi, \tag{23}$$

and transforms homogeneously:

$$\nabla_\mu \Psi \to \mathcal{U} \ \nabla_\mu \Psi, \tag{24}$$

where we are using the matrix notation for the gauge fields $\mathcal{A}_\mu = \sigma^a \mathcal{A}_\mu^a$. Therefore the gauge invariant Lagrangian has the following form:

$$\mathcal{L}^F = \bar{\Psi}\gamma_\mu[i\partial_\mu \Psi + g\mathcal{A}_\mu]\Psi. \tag{25}$$

Expanding this Lagrangian over the vector variable e_λ one can get a series of gauge invariant forms for half-integer fermion fields:

$$\mathcal{L}^F = \sum_{s=0}^\infty f_s \mathcal{L}_{s+1/2}, \tag{26}$$

where f_s are coupling constants. The lower-spin invariant Lagrangian is for the spin-1/2 field:

$$\mathcal{L}_{1/2} = \bar{\psi}^i \gamma_\mu(\delta_{ij} i\partial_\mu \ + \ g\sigma_{ij}^a A_\mu^a)\psi^j = \bar{\psi}(i \ \slashed{\partial} + g \ \slashed{A})\psi \tag{27}$$

and for the spin-vector field ψ_μ together with the additional rank-2 spin-tensor $\psi_{\mu\nu}$ the invariant Lagrangian has the form [4]:

$$\begin{aligned}
\mathcal{L}_{3/2} &= \bar{\psi}_\lambda \gamma_\mu(i\partial_\mu + gA_\mu)\psi_\lambda + \frac{1}{2}\bar{\psi}\gamma_\mu(i\partial_\mu + gA_\mu)\psi_{\lambda\lambda} + \frac{1}{2}\bar{\psi}_{\lambda\lambda}\gamma_\mu(i\partial_\mu + gA_\mu)\psi \\
&+ g\bar{\psi}_\lambda \gamma_\mu A_{\mu\lambda}\psi + g\bar{\psi}\gamma_\mu A_{\mu\lambda}\psi_\lambda + \frac{1}{2}g\bar{\psi}\gamma_\mu A_{\mu\lambda\lambda}\psi \ ,
\end{aligned} \tag{28}$$

and it is invariant under simultaneous extended gauge transformations of the fermions (22) and tensor gauge fields (7):

$$\delta\mathcal{L}_{3/2} = 0.$$

The currents are given by the variation of the action over the tensor gauge fields:

$$\begin{aligned}
J_\mu^a &= g\{\bar{\psi}_\lambda \sigma^a \gamma_\mu \psi_\lambda + \frac{1}{2}\bar{\psi}_{\lambda\lambda}\sigma^a \gamma_\mu \psi + \frac{1}{2}\bar{\psi}\sigma^a \gamma_\mu \psi_{\lambda\lambda}\}, \\
J_{\mu\nu}^a &= g\{\bar{\psi}\sigma^a \gamma_\mu \psi_\nu + \bar{\psi}_\nu \sigma^a \gamma_\mu \psi\}, \\
J_{\mu\lambda\rho}^a &= \frac{1}{2}g\bar{\psi}\sigma^a \gamma_\mu \psi \ \eta_{\lambda\rho}.
\end{aligned} \tag{29}$$

From extended gauge invariance it follows that they are divergenceless with respect to the first indices:

$$\partial_\mu J_\mu^a = \partial_\mu J_{\mu\nu}^a = \partial_\mu J_{\mu\lambda\rho}^a = 0. \tag{30}$$

In the next section we shall see that there exists a second invariant Lagrangian $\mathcal{L}'_{3/2}$ which can be constructed in terms of these spinor-tensor fields and the total Lagrangian is a linear sum of these two forms $f_1 \mathcal{L}_{3/2} + f'_1 \ \mathcal{L}'_{3/2}$. The coupling constants f_1 and f'_1 remain arbitrary because every term of the sum is separately gauge invariant and the extended gauge symmetry alone does not fix them. There is a freedom to vary these constants without breaking the extended gauge symmetry. We can expect that one can achieve the enhancement of the extended gauge symmetry properly tuning the coupling constants f_1 and f'_1. And indeed, as we shall see, in this way one can achieve the fermion currents conservation with respect to all their indices. This property is necessary in order to have consistent interaction with non-Abelian tensor gauge fields.

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4   The Second Series of Invariant Forms for Fermions

The Lagrangian (28) is not the most general Lagrangian which can be constructed in terms of the above spinor-tensor fields (19). As we shall see, there exists a second invariant Lagrangian $\mathcal{L}'_F$ which can be constructed in terms of spinor-tensor fields (19), and the total Lagrangian is a linear sum of the two Lagrangians: $f \mathcal{L}_F + f' \mathcal{L}'_F$. For the lower-spin case we shall demonstrate that the total Lagrangian $f_1 \mathcal{L}_{3/2} + f'_1 \mathcal{L}'_{3/2}$ exhibits an enhanced gauge invariance with specially chosen coefficients f'_1.

First, we shall construct general Lagrangian density for arbitrary higher-rank spinor-tensor fields which contains two terms: $f_s \mathcal{L}_{s+1/2} + f'_s \mathcal{L}'_{s+1/2}$, s=1,2,... Indeed, let us consider the gauge invariant tensor density of the form [3, 5]

$$\mathcal{L}_{\rho_1\rho_2} = \bar{\Psi}(x,e)\gamma_{\rho_1}[i\partial_{\rho_2} + g\sigma^a \mathcal{A}^a_{\rho_2}(x,e)]\Psi(x,e). \tag{31}$$

It is gauge invariant tensor density because its variation is equal to zero:

$$\delta\mathcal{L}_{\rho_1\rho_2}(x,e) = i\bar{\Psi}(x,e)\xi(x,e)\gamma_{\rho_1}[i\partial_{\rho_2} + g\mathcal{A}_{\rho_2}(x,e)]\Psi(x,e) +$$
$$+\bar{\Psi}(x,e)\gamma_{\rho_1}g(-\frac{1}{g})[\partial_{\rho_2}\xi(x,e) - ig[\mathcal{A}_{\rho_2}(x,e),\xi(x,e)]\Psi(x,e) +$$
$$-i\bar{\Psi}(x,e)\gamma_{\rho_1}[i\partial_{\rho_2} + g\sigma^a \mathcal{A}^a_{\rho_2}(x,e)]\xi(x,e)\Psi(x,e) = 0,$$

where $\mathcal{A}_{\rho_2}(x,e) = \sigma^a \mathcal{A}^a_{\rho_2}(x,e)$. The Lagrangian density (31) generates the series of *gauge invariant tensor densities* $(\mathcal{L}'_{\rho_1\rho_2})_{\lambda_1...\lambda_s}(x)$, when we expand it in powers of the vector variable e:

$$\mathcal{L}_{\rho_1\rho_2}(x,e) = \sum_{s=0}^{\infty} (\mathcal{L}'_{\rho_1\rho_2})_{\lambda_1...\lambda_s}(x) \, e_{\lambda_1}...e_{\lambda_s}. \tag{32}$$

The gauge invariant tensor densities $(\mathcal{L}'_{\rho_1\rho_2})_{\lambda_1...\lambda_s}(x)$ allow to construct two series of gauge invariant Lagrangians: $\mathcal{L}_{s+1/2}$ and $\mathcal{L}'_{s+1/2}$, s=1,2,.. by the contraction of the corresponding tensor indices.

The lower gauge invariant tensor density has the form

$$(\mathcal{L}_{\rho_1\rho_2})_{\lambda_1\lambda_2} = \frac{1}{2}\{ + \bar{\psi}_{\lambda_1}\gamma_{\rho_1}[i\partial_{\rho_2} + gA_{\rho_2}]\psi_{\lambda_2} + \bar{\psi}_{\lambda_2}\gamma_{\rho_1}[i\partial_{\rho_2} + gA_{\rho_2}]\psi_{\lambda_1} +$$
$$+ \bar{\psi}_{\lambda_1\lambda_2}\gamma_{\rho_1}[i\partial_{\rho_2} + gA_{\rho_2}]\psi + \bar{\psi}\gamma_{\rho_1}[i\partial_{\rho_2} + gA_{\rho_2}]\psi_{\lambda_1\lambda_2} +$$
$$+ g\bar{\psi}_{\lambda_1}\gamma_{\rho_1}A_{\rho_2\lambda_2}\psi + g\bar{\psi}_{\lambda_2}\gamma_{\rho_1}A_{\rho_2\lambda_1}\psi +$$
$$+ g\bar{\psi}\gamma_{\rho_1}A_{\rho_2\lambda_2}\psi_{\lambda_1} + g\bar{\psi}\gamma_{\rho_1}A_{\rho_2\lambda_1}\psi_{\lambda_2} + g\bar{\psi}\gamma_{\rho_1}A_{\rho_2\lambda_1\lambda_2}\psi\}, \tag{33}$$

and we shall use it to generate Lorentz invariant densities. Performing contraction of the indices of this tensor density with respect to $\eta_{\rho_1\rho_2}\eta_{\lambda_1\lambda_2}$ we shall reproduce our first gauge invariant Lagrangian density $\mathcal{L}_{3/2}$ (28) presented in the previous section. We shall get the second gauge invariant Lagrangian performing the contraction with respect to the $\eta_{\rho_1\lambda_1}\eta_{\rho_2\lambda_2}$, which is obviously different form the previous one:

$$\mathcal{L}'_{3/2} = \frac{1}{2}\{\bar{\psi}_\mu\gamma_\mu(i\partial_\lambda + gA_\lambda)\psi_\lambda + \bar{\psi}_\lambda(i\partial_\lambda + gA_\lambda)\gamma_\mu\psi_\mu +$$
$$+ \bar{\psi}_{\mu\lambda}\gamma_\mu(i\partial_\lambda + gA_\lambda)\psi + \bar{\psi}(i\partial_\lambda + gA_\lambda)\gamma_\mu\psi_{\mu\lambda} + \tag{34}$$
$$+ g\bar{\psi}_\mu\gamma_\lambda A_{\mu\lambda}\psi + g\bar{\psi}\gamma_\mu A_{\lambda\mu}\psi_\lambda + g\bar{\psi}_\mu\gamma_\mu A_{\lambda\lambda}\psi + g\bar{\psi}\gamma_\mu A_{\lambda\lambda}\psi_\mu + g\bar{\psi}\gamma_\mu A_{\lambda\mu\lambda}\psi \}.$$

One can also prove independently from the above consideration, that these Lagrangian forms are invariant under simultaneous extended gauge transformations of fermions (22) and tensor gauge fields (7), calculating their variation:

$$\delta \mathcal{L}'_{3/2} = 0.$$

The currents are given by the variation of the action over the tensor gauge fields:

$$
\begin{aligned}
J'^{a}_{\mu} &= \frac{1}{2} g \{ \bar{\psi}_{\mu} \sigma^{a} \gamma_{\lambda} \psi_{\lambda} + \bar{\psi}_{\lambda} \sigma^{a} \gamma_{\lambda} \psi_{\mu} + \bar{\psi}_{\mu\lambda} \sigma^{a} \gamma_{\lambda} \psi + \bar{\psi} \sigma^{a} \gamma_{\lambda} \psi_{\mu\lambda} \}, & (35) \\
J'^{a}_{\mu\nu} &= \frac{1}{2} g \{ \bar{\psi}_{\mu} \sigma^{a} \gamma_{\nu} \psi + \bar{\psi} \sigma^{a} \gamma_{\nu} \psi_{\mu} + \eta_{\mu\nu} (\bar{\psi}_{\lambda} \sigma^{a} \gamma_{\lambda} \psi + \bar{\psi} \sigma^{a} \gamma_{\lambda} \psi_{\lambda}) \}, \\
J'^{a}_{\mu\lambda\rho} &= \frac{1}{4} g (\bar{\psi} \sigma^{a} \gamma_{\lambda} \psi \; \eta_{\mu\rho} + \bar{\psi} \sigma^{a} \gamma_{\rho} \psi \; \eta_{\mu\lambda}).
\end{aligned}
$$

From extended gauge invariance it follows that they are divergenceless with respect to the first indices:

$$\partial_{\mu} J'^{a}_{\mu} = \partial_{\mu} J'^{a}_{\mu\nu} = \partial_{\mu} J'^{a}_{\mu\lambda\rho} = 0. \tag{36}$$

Thus the total Lagrangian is a linear sum of the two Lagrangians: $f_1 \mathcal{L}_{3/2} + f'_1 \mathcal{L}'_{3/2}$. As one can see, from the Lagrangians (28) and (34) the interaction of fermions with tensor gauge bosons is going through the cubic vertex which includes two fermions and a tensor gauge boson, very similar to the vertices in QED and the Yang-Mills theory.

5   Euler-Lagrange Equations and Enhanced Symmetry

As we found above, the total Lagrangian is a linear sum of the two Lagrangians $f_1 \mathcal{L}_{3/2} + f'_1 \mathcal{L}'_{3/2} = f_1 (\mathcal{L}_{3/2} + d_1 \mathcal{L}'_{3/2})$ and has the form

$$
\begin{aligned}
\mathcal{L}_{3/2} + d_1 \mathcal{L}'_{3/2} &= \bar{\psi}_{\lambda} \gamma_{\mu} (i\partial_{\mu} + g A_{\mu}) \psi_{\lambda} + \frac{1}{2} \bar{\psi} \gamma_{\mu} (i\partial_{\mu} + g A_{\mu}) \psi_{\lambda\lambda} + \frac{1}{2} \bar{\psi}_{\lambda\lambda} \gamma_{\mu} (i\partial_{\mu} + g A_{\mu}) \psi \\
&+ g \bar{\psi}_{\lambda} \gamma_{\mu} A_{\mu\lambda} \psi + g \bar{\psi} \gamma_{\mu} A_{\mu\lambda} \psi_{\lambda} + \frac{1}{2} g \bar{\psi} \gamma_{\mu} A_{\mu\lambda\lambda} \psi + & (37) \\
&+ d_1 \frac{1}{2} \{ \bar{\psi}_{\mu} \gamma_{\mu} (i\partial_{\lambda} + g A_{\lambda}) \psi_{\lambda} + \bar{\psi}_{\lambda} (i\partial_{\lambda} + g A_{\lambda}) \gamma_{\mu} \psi_{\mu} + \\
&+ \bar{\psi}_{\mu\lambda} \gamma_{\mu} (i\partial_{\lambda} + g A_{\lambda}) \psi + \bar{\psi} (i\partial_{\lambda} + g A_{\lambda}) \gamma_{\mu} \psi_{\mu\lambda} + \\
&+ g \bar{\psi}_{\mu} \gamma_{\lambda} A_{\mu\lambda} \psi + g \bar{\psi} \gamma_{\mu} A_{\lambda\mu} \psi_{\lambda} + g \bar{\psi}_{\mu} \gamma_{\mu} A_{\lambda\lambda} \psi + g \bar{\psi} \gamma_{\mu} A_{\lambda\lambda} \psi_{\mu} + g \bar{\psi} \gamma_{\mu} A_{\lambda\mu\lambda} \psi \}.
\end{aligned}
$$

Our aim now is to find out, if there exists a linear combination of these forms which will produce higher symmetry of the total Lagrangian. In the weak coupling limit $g \to 0$ it will take the form

$$
\begin{aligned}
\mathcal{L}_{3/2} + d_1 \mathcal{L}'_{3/2} &= \bar{\psi}_{\lambda} \gamma_{\mu} i\partial_{\mu} \psi_{\lambda} + \frac{1}{2} \bar{\psi} \gamma_{\mu} i\partial_{\mu} \psi_{\lambda\lambda} + \frac{1}{2} \bar{\psi}_{\lambda\lambda} \gamma_{\mu} i\partial_{\mu} \psi \\
&+ \frac{d_1}{2} \{ \bar{\psi}_{\mu} \gamma_{\mu} i\partial_{\lambda} \psi_{\lambda} + \bar{\psi}_{\lambda} i\partial_{\lambda} \gamma_{\mu} \psi_{\mu} + \bar{\psi}_{\mu\lambda} \gamma_{\mu} i\partial_{\lambda} \psi + \bar{\psi} i\partial_{\lambda} \gamma_{\mu} \psi_{\mu\lambda} \}.
\end{aligned}
$$

We have the following free equations of motion:

$$\gamma_{\mu} i\partial_{\mu} \psi + \frac{1}{2} \gamma_{\mu} i\partial_{\mu} \psi_{\lambda\lambda} + d_1 \frac{1}{4} (\gamma_{\mu} i\partial_{\lambda} + \gamma_{\lambda} i\partial_{\mu}) \psi_{\mu\lambda} = 0$$

$$\gamma_\lambda i\partial_\lambda\psi_\mu + d_1\frac{1}{2}(\gamma_\mu i\partial_\lambda + \gamma_\lambda i\partial_\mu)\psi_\lambda = 0 \tag{38}$$

$$\eta_{\mu\lambda}\gamma_\rho i\partial_\rho\psi + d_1\frac{1}{2}(\gamma_\mu i\partial_\lambda + \gamma_\lambda i\partial_\mu)\psi = 0$$

or, in equivalent form:

$$\not{p}\psi + \frac{1}{2}\not{p}\psi_{\lambda\lambda} + d_1\,\frac{1}{4}(\gamma_\mu p_\lambda + \gamma_\lambda p_\mu)\psi_{\mu\lambda} = 0$$

$$\not{p}\psi_\mu + d_1\frac{1}{2}(\gamma_\mu p_\lambda + \gamma_\lambda p_\mu)\psi_\lambda = 0 \tag{39}$$

$$\eta_{\mu\lambda}\,\not{p}\psi + d_1\frac{1}{2}(\gamma_\mu p_\lambda + \gamma_\lambda p_\mu)\psi = 0,$$

where $\not{p} = \gamma_\mu p_\mu = \gamma_\mu i\partial_\mu$. The corresponding total currents $J^{tot} = J + J'$ are equal to the sum of (29) and (35). Calculating the derivatives of these currents and using equations of motion one can see, that the conservation of the total currents over all indices takes place, when $d_1 = 2$, thus

$$
\begin{aligned}
J_\mu^{tot\ a} &= g(\bar{\psi}_\lambda\sigma^a\gamma_\mu\psi_\lambda + \frac{1}{2}\bar{\psi}_{\lambda\lambda}\sigma^a\gamma_\mu\psi + \frac{1}{2}\bar{\psi}\sigma^a\gamma_\mu\psi_{\lambda\lambda} + \\
&\quad + \bar{\psi}_\mu\sigma^a\gamma_\lambda\psi_\lambda + \bar{\psi}_\lambda\sigma^a\gamma_\lambda\psi_\mu + \bar{\psi}_{\mu\lambda}\sigma^a\gamma_\lambda\psi + \bar{\psi}\sigma^a\gamma_\lambda\psi_{\mu\lambda}), \\
J_{\mu\nu}^{tot\ a} &= g(\bar{\psi}\sigma^a\gamma_\mu\psi_\nu + \bar{\psi}_\nu\sigma^a\gamma_\mu\psi + \\
&\quad + \bar{\psi}_\mu\sigma^a\gamma_\nu\psi + \bar{\psi}\sigma^a\gamma_\nu\psi_\mu + \eta_{\mu\nu}(\bar{\psi}_\lambda\sigma^a\gamma_\lambda\psi + \bar{\psi}\sigma^a\gamma_\lambda\psi_\lambda)), \\
J_{\mu\lambda\rho}^{tot\ a} &= \frac{1}{2}g(\bar{\psi}\sigma^a\gamma_\mu\psi\ \eta_{\lambda\rho} + \bar{\psi}\sigma^a\gamma_\lambda\psi\ \eta_{\mu\rho} + \bar{\psi}\sigma^a\gamma_\rho\psi\ \eta_{\mu\lambda}),
\end{aligned}
\tag{40}
$$

and we have conservation of the total tensor currents over all indices:

$$
\begin{aligned}
\partial_\nu J_\nu^{tot\ a} &= 0, \\
\partial_\nu J_{\nu\lambda}^{tot\ a} &= 0, \quad \partial_\lambda J_{\nu\lambda}^{tot\ a} = 0, \\
\partial_\nu J_{\nu\lambda\rho}^{tot\ a} &= 0, \quad \partial_\lambda J_{\nu\lambda\rho}^{tot\ a} = 0, \quad \partial_\rho J_{\nu\lambda\rho}^{tot\ a} = 0.
\end{aligned}
\tag{41}
$$

This result is essential for the consistency of the interaction between tensor gauge bosons and fermions.

It is remarkable, that for a different choice of the coefficient d_1 a new type of gauge symmetry arises. Let us consider the gauge transformation of the spinors-tensor fields of the Rarita-Schwinger-Fang-Fronsdal form:

$$
\begin{aligned}
\delta\psi &= 0 \\
\delta\psi_{\lambda_1} &= \partial_{\lambda_1}\varepsilon \\
\delta\psi_{\lambda_1\lambda_2} &= \partial_{\lambda_1}\varepsilon_{\lambda_2} + \partial_{\lambda_2}\varepsilon_{\lambda_1} \\
\delta\psi_{\lambda_1\lambda_2\lambda_3} &= \partial_{\lambda_1}\varepsilon_{\lambda_2\lambda_3} + \partial_{\lambda_2}\varepsilon_{\lambda_1\lambda_3} + \partial_{\lambda_3}\varepsilon_{\lambda_1\lambda_2} \\
.... &= ....
\end{aligned}
\tag{42}
$$

The variation of the first equation in (39) will take the form

$$\not{p}\partial_\lambda\varepsilon_\lambda + d_1\,\frac{1}{4}(2\,\not{p}\partial_\lambda\varepsilon_\lambda + 2i\partial^2\gamma_\lambda\varepsilon_\lambda)$$

and, if we chose $d_1 = -2$ and shall limit the spinor-tensor parameter ε_λ to fulfil the traceless condition

$$\gamma_\lambda \varepsilon_\lambda = 0,$$

the first equation will remain unchanged. As one can clearly see, the rest of the equations in (39) are also invariant with respect to the RSFF transformation, if spinor-tensor parameters $\varepsilon_{\lambda_1 \ldots \lambda_{s-1}}$ fulfil the traceless conditions:

$$\gamma_\lambda \partial_\lambda \varepsilon = 0, \qquad \gamma_{\lambda_1} \varepsilon_{\lambda_1 \lambda_2 \ldots \lambda_{s-1}} = 0, \qquad s = 2, 3, \ldots \tag{43}$$

In this case the tensor currents will take the form

$$
\begin{aligned}
J^{tot\ a}_\mu &= g(\bar\psi_\lambda \sigma^a \gamma_\mu \psi_\lambda + \frac{1}{2}\bar\psi_{\lambda\lambda}\sigma^a \gamma_\mu \psi + \frac{1}{2}\bar\psi \sigma^a \gamma_\mu \psi_{\lambda\lambda} - \\
&\quad - \bar\psi_\mu \sigma^a \gamma_\lambda \psi_\lambda - \bar\psi_\lambda \sigma^a \gamma_\lambda \psi_\mu - \bar\psi_{\mu\lambda}\sigma^a \gamma_\lambda \psi - \bar\psi \sigma^a \gamma_\lambda \psi_{\mu\lambda}), \\
J^{tot\ a}_{\mu\nu} &= g(\bar\psi \sigma^a \gamma_\mu \psi_\nu + \bar\psi_\nu \sigma^a \gamma_\mu \psi - \\
&\quad - \bar\psi_\mu \sigma^a \gamma_\nu \psi - \bar\psi \sigma^a \gamma_\nu \psi_\mu - \eta_{\mu\nu}(\bar\psi_\lambda \sigma^a \gamma_\lambda \psi + \bar\psi \sigma^a \gamma_\lambda \psi_\lambda)), \\
J^{tot\ a}_{\mu\lambda\rho} &= \frac{1}{2}g(\bar\psi \sigma^a \gamma_\mu \psi\ \eta_{\lambda\rho} - \bar\psi \sigma^a \gamma_\lambda \psi\ \eta_{\mu\rho} - \bar\psi \sigma^a \gamma_\rho \psi\ \eta_{\mu\lambda}).
\end{aligned}
\tag{44}
$$

Corresponding fermion currents are not divergence free, but only traceless part of the divergence vanishes [20, 21, 22].

6 *The First Gauge Invariant Lagrangian for Bosons*

We are in a position now to introduce the gauge invariant interaction of the tensor gauge bosons with the boson field $\phi_{\lambda_1 \ldots \lambda_s}(x)$. This set of tensor fields $\{\phi\}$ contains the scalar field ϕ as one of its family members. The extended isotopic transformation of the bosonic matter fields $\phi_{\lambda_1 \ldots \lambda_s}(x)$ we shall define by the formulas [3, 4, 5]

$$
\begin{aligned}
\delta_\xi \phi &= -i\tau^a \xi^a \phi, \\
\delta_\xi \phi_\lambda &= -i\tau^a (\xi^a\ \phi_\lambda + \xi^a_\lambda\ \phi), \\
\delta_\xi \phi_{\lambda\rho} &= -i\tau^a (\xi^a\ \phi_{\lambda\rho} + \xi^a_\lambda\ \phi_\rho + \xi^a_\rho\ \phi_\lambda + \xi^a_{\lambda\rho}\ \phi), \\
&\quad \ldots\ldots \quad . \quad \ldots\ldots\ldots\ldots,
\end{aligned}
\tag{45}
$$

where τ^a are the matrices of the representation τ of the compact Lie group G, according to which the whole family of $\phi's$ transforms. There is an essential difference in the transformation properties of the tensor gauge fields $A_{\mu\lambda_1 \ldots \lambda_s}$ versus $\phi_{\lambda_1 \ldots \lambda_s}$. The transformation law for the bosonic matter fields (45) is homogeneous, whereas the transformation of the tensor gauge fields (7) is inhomogeneous. The general form of the above transformation is:

$$\delta_\xi \phi_{\lambda_1 \ldots \lambda_s}(x) = -i \sum_{i=0}^{s} \sum_{P's} \xi_{\lambda_1 \ldots \lambda_i}\ \phi_{\lambda_{i+1} \ldots \lambda_s}(x), \qquad s = 0, 1, 2, \ldots \quad , \tag{46}$$

and the invariant quadratic form is:

$$U(\phi) = \sum_{s=0}^{\infty} \lambda_{s+1} U_s(\phi), \qquad U_s(\phi) = \sum_{i=0}^{2s} a^s_i\ \phi^+_{\lambda_1 \ldots \lambda_i}\ \phi_{\lambda_{i+1} \ldots \lambda_{2s}} \sum_{p's} \eta^{\lambda_{i_1} \lambda_{i_2}} \ldots\ldots \eta^{\lambda_{i_{2s-1}} \lambda_{i_{2s}}}\ , \tag{47}$$

where λ_s are arbitrary coupling constants and the sum $\sum_p$ runs over all permutations of $p's$ and the numerical coefficient $a_i^s = s!/i!(2s-i)!$, $\lambda_1 = 1$. Notice that the number of real gauge parameters $\xi_{\lambda_1...\lambda_s}^a$ is proportional to the dimension dimG of the compact Lie group G, while the number of tensor matter fields $\phi_{\lambda_1...\lambda_s}^i$ is proportional to the dimension of the representation τ_{ij}^a of the group G. Because they are totally symmetric tensors, they have the same space-time dimensions, thus

$$dim\xi = dimG \times \ dimT, \quad dim\phi = dim\tau \times \ dimT,$$

where $dimT$ is the dimension of the totally symmetric rank-s tensor.

The invariant Lagrangian for scalar field is

$$\mathcal{L}_0^B = -\nabla_\mu^{ij}\phi^{+j}\ \nabla_\mu^{ik}\phi^k\ - U(\phi),$$

where $\nabla_\mu^{ij} = \delta^{ij}\partial_\mu - ig\tau_a^{ij}A_\mu^a$ and for the rank-one field it has the form [3, 4, 5]:

$$
\begin{aligned}
-\mathcal{L}_1^B &= \nabla_\mu\phi_\lambda^+\ \nabla_\mu\phi_\lambda + \frac{1}{2}\nabla_\mu\phi_{\lambda\lambda}^+\ \nabla_\mu\phi + \frac{1}{2}\nabla_\mu\phi^+\ \nabla_\mu\phi_{\lambda\lambda} + \\
&- ig\nabla_\mu\phi^+\ A_{\mu\lambda}\phi_\lambda + ig\phi_\lambda^+ A_{\mu\lambda}\ \nabla_\mu\phi - ig\nabla_\mu\phi_\lambda^+\ A_{\mu\lambda}\phi + ig\phi^+ A_{\mu\lambda}\ \nabla_\mu\phi_\lambda + \\
&+ g^2\phi^+ A_{\mu\lambda}A_{\mu\lambda}\phi - \frac{1}{2}ig\nabla_\mu\phi^+\ A_{\mu\lambda\lambda}\phi + \frac{1}{2}ig\phi^+ A_{\mu\lambda\lambda}\ \nabla_\mu\phi\ +\ U(\phi). \quad (48)
\end{aligned}
$$

The variation of the Lagrangian is equal to zero, $\delta\mathcal{L}_1^B = 0$. In the next section we shall see, that there exists a second invariant form which can be constructed in terms of boson fields and the total Lagrangian is a linear sum.

7 *The Second Gauge Invariant Lagrangian for Bosons*

The Lagrangian (48) is not the most general Lagrangian which can be constructed in terms of the above boson fields $\phi_{\lambda_1...\lambda_s}(x)$. As we shall see, here also exists a second invariant form $\mathcal{L}_1'^B$ which can be constructed in terms of boson fields $\phi_{\lambda_1...\lambda_s}(x)$ and the total Lagrangian is a linear sum of them: $b_1\mathcal{L}_1^B + b_1'\mathcal{L}_1'^B$. The sum exhibits additional gauge invariance with specially chosen coefficient b_1' .

Let us consider the gauge invariant tensor density of the form [3, 4, 5]

$$\mathcal{L}_{\rho_1\rho_2} = \nabla_{\rho_1}^{ij}(e)\Phi^{+j}(x,e)\ \nabla_{\rho_2}^{ik}(e)\Phi^k(x,e), \quad (49)$$

where $\nabla_\mu^{ij}(e) = \delta^{ij}\partial_\mu - ig\tau_a^{ij}\mathcal{A}_\mu^a(x,e)$. The Lagrangian density (49) generates the second series of *gauge invariant tensor densities* $(\mathcal{L}_{\rho_1\rho_2})_{\lambda_1...\lambda_s}(x)$, when we expand it in powers of the vector variable e_λ:

$$\mathcal{L}_{\rho_1\rho_2}(x,e) = \sum_{s=0}^\infty\ (\mathcal{L}_{\rho_1\rho_2})_{\lambda_1...\lambda_s}(x)\ e_{\lambda_1}...e_{\lambda_s}. \quad (50)$$

The lower gauge invariant tensor density has the form

$$
\begin{aligned}
(\mathcal{L}_{\rho_1\rho_2})_{\lambda_1\lambda_2} &= \\
&+ \frac{1}{2}\{\ \nabla_{\rho_1}\phi_{\lambda_1}^+\nabla_{\rho_2}\phi_{\lambda_2} + \nabla_{\rho_1}\phi_{\lambda_2}^+\nabla_{\rho_2}\phi_{\lambda_1} + \nabla_{\rho_1}\phi_{\lambda_1\lambda_2}^+\nabla_{\rho_2}\phi + \nabla_{\rho_1}\phi^+\nabla_{\rho_2}\phi_{\lambda_1\lambda_2} \quad (51) \\
&- ig\nabla_{\rho_1}\phi_{\lambda_1}^+ A_{\rho_2\lambda_2}\phi - ig\nabla_{\rho_1}\phi_{\lambda_2}^+ A_{\rho_2\lambda_1}\phi - ig\nabla_{\rho_1}\phi^+ A_{\rho_2\lambda_1}\phi_{\lambda_2} - ig\nabla_{\rho_1}\phi^+ A_{\rho_2\lambda_2}\phi_{\lambda_1} \\
&+ ig\phi_{\lambda_1}^+ A_{\rho_1\lambda_2}\nabla_{\rho_2}\phi + ig\phi_{\lambda_2}^+ A_{\rho_1\lambda_1}\nabla_{\rho_2}\phi + ig\phi^+ A_{\rho_1\lambda_1}\nabla_{\rho_2}\phi_{\lambda_2} + ig\phi^+ A_{\rho_1\lambda_2}\nabla_{\rho_2}\phi_{\lambda_1} \\
&+ g^2\phi^+ A_{\rho_1\lambda_1}A_{\rho_2\lambda_2}\phi + g^2\phi^+ A_{\rho_1\lambda_2}A_{\rho_2\lambda_1}\phi - ig\nabla_{\rho_1}\phi^+ A_{\rho_2\lambda_1\lambda_2}\phi + ig\phi^+ A_{\rho_1\lambda_1\lambda_2}\nabla_{\rho_2}\phi\},
\end{aligned}
$$

and by an appropriate contraction of indices generates Lorentz invariant densities. Performing contraction of the indices of this tensor density with respect to $\eta_{\rho_1\rho_2}\eta_{\lambda_1\lambda_2}$ we shall get our first gauge invariant Lagrangian density $\mathcal{L}_1^B$ (48) presented in the previous section.

We shall get the second gauge invariant Lagrangian performing the contraction with respect to $\eta_{\rho_1\lambda_1}\eta_{\rho_2\lambda_2}$, which is obviously different from the previous one:

$$
\begin{aligned}
\mathcal{L}_1'^B \;=\; \frac{1}{2}\{ & \nabla_\mu\phi_\mu^+\nabla_\lambda\phi_\lambda + \nabla_\mu\phi_\lambda^+\nabla_\lambda\phi_\mu + \nabla_\mu\phi_{\mu\lambda}^+\nabla_\lambda\phi + \nabla_\mu\phi^+\nabla_\lambda\phi_{\mu\lambda} - \\
& -ig\nabla_\mu\phi_\mu^+A_{\lambda\lambda}\phi - ig\nabla_\mu\phi_\lambda^+A_{\lambda\mu}\phi - ig\nabla_\mu\phi^+A_{\lambda\mu}\phi_\lambda - ig\nabla_\mu\phi^+A_{\lambda\lambda}\phi_\mu + \\
& +ig\phi_\mu^+A_{\lambda\lambda}\nabla_\mu\phi + ig\phi_\mu^+A_{\mu\lambda}\nabla_\lambda\phi + ig\phi^+A_{\mu\mu}\nabla_\lambda\phi_\lambda + ig\phi^+A_{\mu\lambda}\nabla_\lambda\phi_\mu + \\
& +g^2\phi^+A_{\mu\mu}A_{\lambda\lambda}\phi + g^2\phi^+A_{\mu\lambda}A_{\lambda\mu}\phi - ig\nabla_\mu\phi^+A_{\lambda\mu\lambda}\phi + ig\phi^+A_{\lambda\mu\lambda}\nabla_\mu\phi \}.
\end{aligned}
\tag{52}
$$

Thus the total Lagrangian $\mathcal{L}_0^B + b(\mathcal{L}_1^B + h\mathcal{L}_1'^B)$ has a linear sum of two forms and our aim is to find out, if there exists a linear combination of these forms which admits additional higher symmetries. In zero coupling limit it will take the form

$$
\begin{aligned}
\mathcal{L}_1^B + h\mathcal{L}_1'^B \;=\; & -\partial_\mu\phi_\lambda^+\,\partial_\mu\phi_\lambda + \frac{h}{2}\{\,\partial_\mu\phi_\mu^+\partial_\lambda\phi_\lambda + \partial_\mu\phi_\lambda^+\partial_\lambda\phi_\mu\} - \\
& -\frac{1}{2}\partial_\mu\phi_{\lambda\lambda}^+\,\partial_\mu\phi - \frac{1}{2}\partial_\mu\phi^+\,\partial_\mu\phi_{\lambda\lambda} + \frac{h}{2}\{\,\partial_\mu\phi_{\mu\lambda}^+\partial_\lambda\phi + \partial_\mu\phi^+\partial_\lambda\phi_{\mu\lambda}\}.
\end{aligned}
$$

If we take $h = 1$, it will reduce to the form

$$
\begin{aligned}
\mathcal{L}_1^B + \mathcal{L}_1'^B \;=\; & -\partial_\mu\phi_\lambda^+\,\partial_\mu\phi_\lambda + \partial_\mu\phi_\lambda^+\partial_\lambda\phi_\mu - \\
& -\frac{1}{2}\partial_\mu\phi_{\lambda\lambda}^+\,\partial_\mu\phi - \frac{1}{2}\partial_\mu\phi^+\,\partial_\mu\phi_{\lambda\lambda} + \frac{1}{2}\partial_\mu\phi_{\mu\lambda}^+\partial_\lambda\phi + \frac{1}{2}\partial_\mu\phi^+\partial_\lambda\phi_{\mu\lambda}
\end{aligned}
\tag{53}
$$

and become invariant with respect to the gauge transformation of the form

$$
\begin{aligned}
\phi &\rightarrow \phi \\
\phi_\mu &\rightarrow \phi_\mu + \partial_\mu\omega \\
\phi_{\mu\nu} &\rightarrow \phi_{\mu\nu} + \partial_\mu\omega_\nu + \partial_\nu\omega_\mu.
\end{aligned}
\tag{54}
$$

This symmetry transformation is an enhanced symmetry of the Lagrangian. The original system was invariant under the gauge transformation (45).

This phenomenon of enhancement of the original symmetries is of the same nature as we have already observed in the case of tensor gauge fields and fermions, where the extended gauge transformation (7) and (22) have been enhanced to larger symmetries (18) and (42). The above enhanced gauge symmetries allow to exclude negative norm states from our system of tensor fields and in the given case the zero component of the boson field ϕ_μ.

8 Symmetry Breaking and Masses of Tensor Gauge Bosons

The Lagrangian $\mathcal{L} = \mathcal{L}_0^B + \mathcal{L}_1^B + \mathcal{L}_1'^B$ can be responsible for the mass generation of the second-rank tensor gauge field $A_{\mu\nu}^a$. The relevant terms have the following form:

$$
\begin{aligned}
\mathcal{L} \;=\; & -\nabla_\mu\phi^+\,\nabla_\mu\phi - U(\phi) \\
& +b_1\{-\nabla_\mu\phi_\lambda^+\,\nabla_\mu\phi_\lambda + \frac{1}{2}\nabla_\mu\phi_\mu^+\nabla_\lambda\phi_\lambda + \frac{1}{2}\nabla_\mu\phi_\lambda^+\nabla_\lambda\phi_\mu \\
& -g^2\phi^+(A_{\mu\lambda}A_{\mu\lambda} - \frac{1}{2}A_{\mu\mu}A_{\lambda\lambda} - \frac{1}{2}A_{\mu\lambda}A_{\lambda\mu})\phi\}.
\end{aligned}
\tag{55}
$$

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The first term describes the standard interaction of the charge vector gauge boson with charged scalar field and with properly chosen scalar potential will generate the mass of the vector boson (see the end of this section). The next three terms describe the interaction of the charged vector gauge bosons A_μ with charged vector boson ϕ_λ and the last three terms describe the interaction of the charged tensor gauge bosons $A_{\mu\nu}$ with charged scalar boson ϕ. When the scalar field gets the vacuum expectation value (62), the charged tensor gauge bosons receive the mass term of the form

$$b_1 g^2 \phi^+ (A_{\mu\lambda} A_{\mu\lambda} - \frac{1}{2} A_{\mu\mu} A_{\lambda\lambda} - \frac{1}{2} A_{\mu\lambda} A_{\lambda\mu}) \phi \rightarrow$$

$$b_1 g^2 < \phi^\dagger \tau_a >< \tau_b \phi > (A_{\mu\lambda}^a A_{\mu\lambda}^b - \frac{1}{2} A_{\mu\mu}^a A_{\lambda\lambda}^b - \frac{1}{2} A_{\mu\lambda}^a A_{\lambda\mu}^b). \tag{56}$$

Decomposing the tensor gauge field $A_{\mu\nu}$ into symmetric and antisymmetric parts $A_{\mu\nu} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu}) + \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu}) = T_{\mu\nu} + S_{\mu\nu}$ one can see that the mass term takes the form

$$\frac{m_T^2}{2}(T_{\mu\lambda} T_{\mu\lambda} - T_{\mu\mu} T_{\lambda\lambda}) + \frac{m_S^2}{2} S_{\mu\lambda} S_{\lambda\mu}, \tag{57}$$

where the mass matrices are

$$m_T^2 = (\frac{b_1}{g_2}) g^2 < \phi^\dagger \tau_a >< \tau_b \phi >, \qquad m_S^2 = 3(\frac{b_1}{g_2}) g^2 < \phi^\dagger \tau_a >< \tau_b \phi >, \tag{58}$$

and we conclude that the coupling constant b_1 should be positive. As we can see from above formulas, the symmetric $T_{\mu\nu}$ and antisymmetric $S_{\mu\nu}$ parts of the tensor gauge field get different masses: the antisymmetric part gets the mass which is three times bigger than that of the symmetric tensor gauge boson. The coupling constant b_1, as we discussed earlier, remains arbitrary in this model, therefore the relation between masses of the tensor gauge bosons and vector gauge bosons $m_V^2 = 2g^2 < \phi^\dagger \tau_a >< \tau_b \phi >$ is given by the relations

$$m_T^2 = (\frac{b_1}{2g_2}) m_V^2, \qquad m_S^2 = 3(\frac{b_1}{2g_2}) m_V^2, \tag{59}$$

with the b_1-*independent* mass ratio

$$\frac{m_S^2}{m_T^2} = 3, \tag{60}$$

which is an interesting prediction of this model[4].

We have to introduce the invariant self-interaction Lagrangian for the extended scalar sector. The first two quadratic forms, which are invariant with respect to the extended homogeneous transformations (45), have the form (47)

$$U(\phi) = \phi^\dagger \phi + \lambda_2 (\phi_\mu^\dagger \phi_\mu + \frac{1}{2} \phi^\dagger \phi_{\mu\mu} + \frac{1}{2} \phi_{\mu\mu}^\dagger \phi). \tag{61}$$

Its invariance can be confirmed by direct calculation similar to the one we performed above. Using this quadratic form we can construct the invariant potential as

$$U(\phi) = \frac{1}{4} \lambda^2 [\phi^\dagger \phi - \eta^2]^2 + \frac{1}{4} \lambda^2 [\phi^\dagger \phi + \lambda_2 (\phi_\lambda^\dagger \phi_\lambda + \frac{1}{2} \phi^\dagger \phi_{\lambda\lambda} + \frac{1}{2} \phi_{\lambda\lambda}^\dagger \phi)]^2 \tag{62}$$

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[4]These mass formulas are written in preposition that $g_2 \neq 1$ in (17), that is, the kinetic term of the tensor gauge field $A_{\mu\nu}$ is normalized to $-(1/4)g_2$.

so that the vacuum expectation value of the scalar field will be as in the standard model:

$$< \phi >_{vac} = \eta/\sqrt{2}.$$

The Higgs boson mass therefore remains the same as in the standard model:

$$m_H = \lambda\eta.$$

The vector boson ϕ_λ can also acquire mass through the interaction term:

$$\frac{1}{4} \lambda^2 \ 2 \ \lambda_2 \ \phi^\dagger\phi \ \phi_\mu^\dagger\phi_\mu \quad \rightarrow \quad \frac{1}{2}\lambda^2\lambda_2 < \phi^\dagger >< \phi > \phi_\mu^\dagger\phi_\mu = \lambda_2\frac{\lambda^2\eta^2}{4}\phi_\mu^\dagger\phi_\mu, \qquad (63)$$

and it is proportional to the mass of the standard Higgs scalar:

$$m_\phi^2 = (\frac{\lambda_2}{4b_1}) \ m_H^2. \qquad (64)$$

We see that λ_2 should be positive. This formula is of the same nature as for the tensor gauge bosons (59) and reflects the fact that masses of higher-spin partners can be expressed through masses of the standard model particles and the coupling constants between them. In the given case these coupling constants are b_1 (48),(52) and λ_2 (47),(62).

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